Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Winter Examination-2015

Subject Name : Problem Solving-I

Subject Code : 5SC03PAE1 **Branch : M. Sc. (Mathematics)**

Semester : 3 **Date : 5 / 12 / 2015** **Time : 2:30 To 5:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions** **(07)**

a. Check whether $u(x,y) = \sinh x \cos y$ is harmonic function or not.

b. Find the fixed points of bilinear transformation $w = \frac{z}{2-z}$.

c. Show that $f(z) = xy + iy$ is not an analytic function.

d. Solve: $D^2y + y = 0$.

e. Show that the matrix $A = \begin{bmatrix} 0 & 100 & -100 \\ 0 & -2 & 100 \\ 0 & 0 & -100 \end{bmatrix}$ is negative semi-definite.

f. Show that if A is an idempotent matrix and $A + B = I$, then B is idempotent and $AB = BA = 0$.

g. Check whether $W = \{(x,y,z)/x,y,z \in Q\}$ is a subspace of R^3 or not.

Q-2 **Attempt all questions** **(14)**

a) The linear operation $L[X]$ is defined by the cross product $L[X] = b \times X$, where $b = [0 \quad 1 \quad 0]^T$ and $X = [x_1 \quad x_2 \quad x_3]^T$ are three dimensional vectors. If the 3×3 matrix M of this equation satisfies $L[X] = M \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ then find the eigen values of M. **[6]**

b) Find the bilinear transformation, which maps the points $z_1 = 2, Z_2 = i$ and $z_3 = -2$ into the points $w_1 = 1, w_2 = i$ and $w_3 = -1$ respectively. **[6]**

c) Find the rank of following $(n+1) \times (n+1)$ matrix, where $a \in R$ **[2]**

$$\begin{bmatrix} 1 & a & \dots & a^n \\ 1 & a & \dots & a^n \\ \vdots & \vdots & \vdots & \vdots \\ 1 & a & \dots & a^n \end{bmatrix}.$$

OR

Q-2 **Attempt all questions** **(14)**

a) Show that the relation $w = \frac{5-4z}{4z-2}$ transforms the circle $|z| = 1$ into a circle of radius unity in the W −plane and find the centre of this circle. **[6]**

b) Show that the system of equations $x - y + 3z = 4, x + z = 2, x + y - z = 0$ has infinitely many solutions. **[6]**

c) Show that the eigen value of a skew-Hermitian matrix is either zero or a purely imaginary number. **[2]**

Q-3 **Attempt all questions** **(14)**

a) Show that for the matrix, **[6]**

$$A = \begin{bmatrix} 1 & 2014 & 2015 \\ 0 & -1 & 2014 \\ 0 & 0 & 1 \end{bmatrix}$$

the sum of algebraic multiplicity is 3 and the sum of geometric multiplicity is 1.

b) Find the eigen values and eigen functions of the Strum-Liouville problem, **[6]**

$$y'' + \lambda y = 0; y(0) = 0, y'(1) = 0.$$

c) Solve: $\frac{dy}{dx} - x\tan(y - x) = 1$. **[2]**

OR

Q-3 **a)** Find and graph the strip $1 < x < 2$ under the mapping $w = \frac{1}{z}$. **[6]**

b) Evaluate: $\int_0^{2+i} z^2 dz$ along the line $y = \frac{x}{2}$. **[6]**

c) Let ξ be a primitive fifth root of unity. Define $A = \begin{pmatrix} \xi^{-1} & 0 & 0 & 0 & 0 \\ 0 & \xi & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \xi & 0 \\ 0 & 0 & 0 & 0 & \xi^2 \end{pmatrix}$. For a vector $V = (v_1, v_2, v_3) \in R^3$ define $|v|_A = \sqrt{|VAV^T}$, where V^T is a transpose of v. If $W = (1, -1, 1, 1, -1)$ then find the value of $|W|_A$. **[2]**

SECTION – II

Q-4 **Attempt the Following questions** **(07)**

a. Define: singular solution.

b. Find the pole of $f(z) = \frac{z-2}{z^2}$.

c. Define: isolated singularity.

d. Show that $f(z) = \int_C \frac{z-3}{z^2-2z+5} dz$, where $C: |z| = 1$ is analytic everywhere within C.

e. Find the dimension of the subspace $W = \{(x_1, x_2, x_3, x_4, x_5)/3x_1 - x_2 + x_3 = 0\}$ of R^5.

f. If M is a 7×5 matrix of rank 3 and N is a 5×7 matrix of rank 5, then find the rank(MN).

g. If A is 3×3 complex Hermitian matrix, which is unitary. Give the distinct eigen values of A.

Q-5 **Attempt all questions** **(14)**

a) Determine all possible Jordan canonical forms for a linear operator $T: V \to V$, whose characteristic polynomial is $\Delta(t) = (t-2)^3(t-5)^2$. **[6]**

b) If $u = e^x(x\cos y - y\sin y)$, then find the analytic function $f(z) = u + iv$. **[6]**

c) If $X = [x_1 \quad x_2 \quad \cdots \quad x_n]^T$ is an n −tuple of non-zero vectors. Find the rank of $n \times n$ matrix $V = X.X^T$. **[2]**

OR

Q-5

a) If $f(z)$ is an analytic function in a domain D and $|f(z)|$ is constant in D then show that $f(z)$ is constant in D. **[6]**

b) By using variation of parameters method, solve: $y'' + y = \sec x$. **[6]**

c) Let A, B and C be real $n \times n$ matrices such that $AB + B^2 = C$ and if C is non-singular then show that $A + B$ and B are non-singular. **[2]**

Q-6 **Attempt all questions** **(14)**

a) Show that **[7]**

$$\int_0^{2\pi} \frac{\cos 2\theta}{1 - 2a\cos\theta + a^2} d\theta = \frac{2\pi a^2}{1 - a^2}, a^2 < 1.$$

b) Find the 3×3 matrix whose eigen values are $1, 2, 3$ and corresponding eigen vectors are $\begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}, \begin{bmatrix}1\\1\\0\end{bmatrix}$. **[7]**

OR

Q-6 **Attempt all Questions**

a) Expand $f(z) = \frac{1}{(z-1)(z-2)}$ in the region $(i)\ |z| < 1,\ (ii)\ 1 < |z| < 2,\ (iii)\ |z| > 2$. **[7]**

b) Show that the matrix **[7]**

$$A = \begin{bmatrix}1 & 2 & 3\\2 & 4 & 6\\3 & 6 & 9\end{bmatrix}$$

is diagonalizable.